ORIE 6365: Continuous Optimization	Spring 2026

Homework 0

Released: Jan 21, 2026 — *Due: Jan 31, 2026*

Instructions

- **Description:** This initial assignment serves as a diagnostic tool to help you assess your preparation for the mathematical rigors of this course. If you find these problems challenging or unfamiliar, it may indicate that the course is more technical than is appropriate for your current backgrounds. Please note that the weight of this homework will be lower (part of the 50% theory homework grade) compared to subsequent assignments.
- **Submission:** Please submit your solutions as a single PDF via Gradescope

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Problem 1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be an arbitrary function that achieves its maximum. Show that:

$$\max_{x \in \mathbb{R}^n} f(x) \quad = \quad -\min_{x \in \mathbb{R}^n} \big[-f(x)\big].$$

Problem 2. Let $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ be an arbitrary continuous function, and $X \subset \mathbb{R}^n$, $Y \subset \mathbb{R}^m$ are two compact sets. Show that:

$$\min_{x \in X} \max_{y \in Y} f(x, y) \quad \geq \quad \max_{y \in Y} \min_{x \in X} f(x, y).$$

Problem 3. Let $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$ are given. Denote by $\|\cdot\|_2$ the standard Euclidean norm, for $y \in \mathbb{R}^m$, $\|y\|_2 := \sqrt{y^\top y}$, and consider the function:

$$f(x) \quad = \quad \tfrac{1}{2}\|Ax - b\|_2^2, \qquad x \in \mathbb{R}^n.$$

Compute the gradient $\nabla f(x)$.

Problem 4. Let $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{m \times m}$. Denote by $\|\cdot\|_F$ the Frobenious norm of a matrix, $\|Y\|_F = \sqrt{\mathrm{tr}(Y^\top Y)}$, and consider the function:

$$f(X) \quad = \quad \tfrac{1}{2}\|AX - B\|_F^2, \qquad X \in \mathbb{R}^{n \times m}.$$

Compute the gradient $\nabla f(X)$.

Problem 5. Let $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{k \times \ell}$ and consider the function:

$$f(X) \quad = \quad \mathrm{tr}(AXB), \qquad X \in \mathbb{R}^{n \times k}.$$

Compute the gradient $\nabla f(X)$.

Problem 6. Let $f : \mathbb{R}^n \to \mathbb{R}$ be several times differentiable function and $x, y \in \mathbb{R}^n$ be fixed.

- Consider a univariate function $g : \mathbb{R} \to \mathbb{R}$ given by $g(t) = f(x + t(y - x))$. Express $g'(t)$ via the gradient of f.
- Using the fundamental theorem of calculus, verify the following integral formula:

$$f(y) - f(x) \quad = \quad \int\limits_0^1 \langle \nabla f(x + t(y - x)), y - x \rangle dt.$$

Problem 7. Let $x^* \in \mathbb{R}^n$ be a fixed point. For any $\varepsilon > 0$, construct a function $f_\varepsilon : \mathbb{R}^n \to \mathbb{R}$ such that

1. f_ε is continuous;
2. $f_\varepsilon(x^*) = 0$;
3. $f_\varepsilon(x) = 1$ for all $x \in C_\varepsilon$, where C_ε is the closed complement of the Euclidean ball:

$$C_\varepsilon \quad = \quad \Big\{ x \in \mathbb{R}^n \; : \; \|x - x^*\|_2 \geq \varepsilon \Big\}.$$

Hint. Consider the case $n = 1$ first.